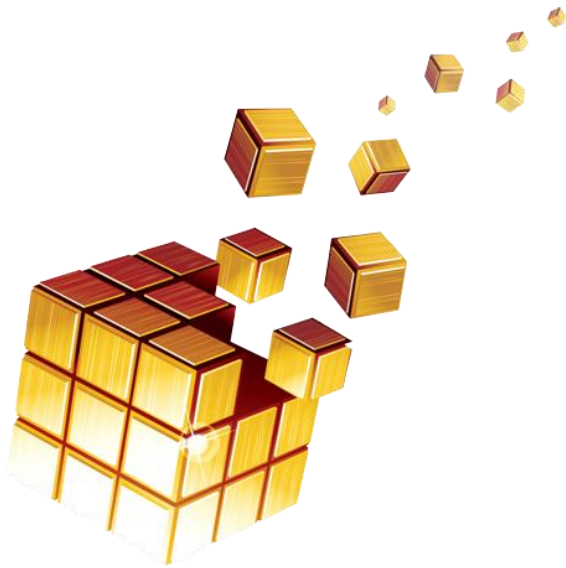
SIM2007 Appreciation of Mathematics

Everything About Rubik's Cube



Group 5

1.	CH'NG JIA QI	U2103224
2.	CHAU PUI KEI	U2103460
3.	GAVIN LIM ZHEN YAO	U2103511
4	KANCHANAA A/P SIVABALAN	U2103645
5.	KUNG SOOK KEAN	U2103458
6.	MEENA A/P MURUGAPPAN ANNAMALAI	U2103652
7.	NG JIA XUAN	U2103227
8.	TAI KAH YIT	U2103464
9.	TENG JIAN HONG	S2132490

Summary

There are numerous Rubik's Cube variations. Some theoretical research and application accomplishments have been made. This report comprehensively introduced the genesis and evolution of Rubik's Cube, studied the structure and functional properties of Rubik's Cube, and assessed the Rubik's Cube research status, including science metaphors, restoration algorithms, and characteristic applications. Since the outward qualities of the Rubik's Cube have been examined and applied in a variety of sectors, the principles underlying the interior structure of the Rubik's Cube was investigated concurrently. It can be believed that Rubik’s Cube will have extensive prospects for applications in the machinery industry based on its research status, and a spill over effect in some scientific research including mathematics, physics, computers, and biology.

Objectives

1. To learn the history and founder of the Rubik’s cubes.
2. To find out the Mathematics theorems about Rubik’s cube.
3. To develop thinking abilities and problem solving skills.
4. To build patience and calmness.
5. To enhance concentration and configuration.

Target community

Our target communities for the project are secondary and university students who are interested to Rubik’s Cube. Our project will be held on 7.1.2023, 8pm using Microsoft Team.

1.0 Introduction

In 1974, a Hungarian sculptor and professor of architecture Ernő Rubik had invented Rubik's cube which is a 3D combination puzzle. Originally, the Rubik's cube is known as the Magic Cube. Due to its distinctive qualities, which had a significant influence on humanity, this invention sparked global curiosity. One of the 100 most important inventions of the 20th century is the Rubik's Cube. It is also often regarded as the toy that sells the most globally. It got a special Game of the Year award in Germany and accolades for best toy in the US, UK, and France.

Even though the Rubik's Cube's widespread popularity peaked in the 1980s, it is still widely recognised and utilised today. Due to its advanced design and concepts, it attracts interest from scientists and technological specialists from a number of sectors. It also attracts Rubik's Cube enthusiasts who do research on Rubik's Cube reduction algorithms. On the one hand, the Rubik's Cube structure has a number of features, including rotation, permutations and combinations, cycle and symmetry, and others, which have been used as tools or physical models to investigate particular scientific problems or, in some cases, were investigated using scientific theory or methods. Overall, many scientific systems that require permutations, combinations, symmetries, and cyclicality have elements of the Rubik's Cube's principles. Hence, we will have a look at some of these Mathematics' theorems about Rubik's cube through this project.

2.0 History Backgroud

2.1 Timeline of the Rubik's Cube

In the spring of 1974, a young Hungarian architect called Ernő Rubik became obsessed with teaching his pupils on how to present three-dimensional movement. After months of working with blocks of cubes constructed of wood and paper which tied together with rubber bands, glue, and paper clips, he finally produced the "Bűvös kocka," or Magic Cube. But two years before Rubik invented his cube, Larry D. Nichols invented a 2x2x2 "Puzzle with Pieces Rotatable in Groups" and filled a Canadian patent application in 1970. In order to hold the parts together, Nichols used magnets as a main mechanism unlike Rubik's Magic Cube.

Then, the Magic Cube's initial test batches were created in late 1977 and sold in Budapest toy stories. With Ernő Rubik's consent, businessman Tibor Laczi ended up taking a Cube to Germany's Nuremberg Toy Fair in February 1979 in an apparent effort to commercialise it. It was noticed by Steven Towns founder Tom Kremer, and in September 1979 they signed a deal with Ideal Toys to release the Magic cube worldwide. After that, the magic cube was renamed as Rubik's Cube and began putting it in stores in 1980. Soon, puzzlers from all over the world were vying to solve the cube. Within two years, they had purchased 100 million of them, making Rubik's Cube the most popular puzzle in history. Its popularity spawned hundreds of spin-off items, ranging from best-selling books on how to solve it to patent-infringing knockoffs by other manufacturers.

2.2 Mechanism of Rubik's Cube

Rubik created the world's first three-order cube, which resembles a sphere and is controlled by restriction between the components to produce a certain rotation. Figure 1 depicts this. Initially, to address security concerns, the cube's eight vertices were shortened somewhat. It was then altered to sharp corners and became a cubic form since its vertices did not affect the aesthetic and the manufacturing process became easier. Figure 2 depicts today's common third-order cube. It can be observed that the form of Rubik's Cube may be changed without altering the structure of the cube's rotation.

Figure 1: Early Rubik's Cube

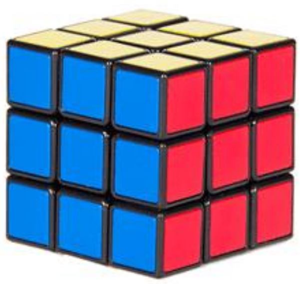


Figure 2: Current Rubik's Cube

Following the invention of the Rubik's Cube, several modified cubes were created to expand the cube family. In general, there are two kinds of magic cubes: cubic cubes and specifically shaped cubes. A cubic cube is a cube in a box structure whose order grows. There are several Rubik's cube varieties with up to 33 layers, with the most widely known being the 2x2x2 (Pocket/Mini Cube), standard 3x3x3 cube, 4x4x4 (Rubik's Revenge/ Master Cube), and 5x5x5 (Professor's Cube). Until December 2017, the largest and the most costly, (costing more than US $2,000) commercially offered cube was the 17x17x17 (Over The Top). Moving on, there is a workable design for the 22x22x22 cube, which was exhibited in January 2016, and a 33x33x33 cube was revealed in December 2017. Figure 3 lists all the six cases of the cubes.

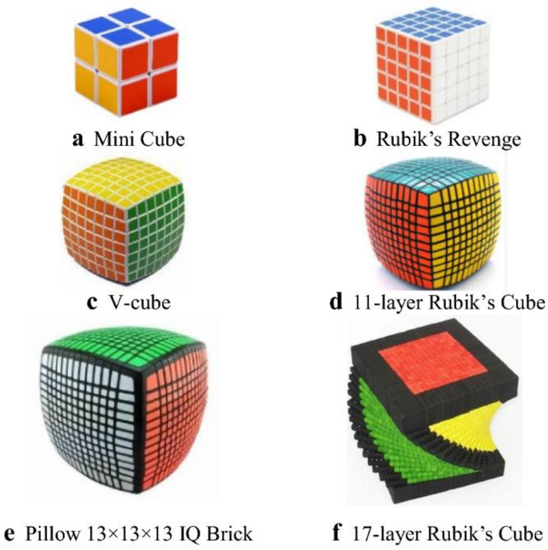


Figure 3: Models of Cubic Rubik's Cube

Aside from the cubic cube, the term "specifically formed cube" refers to the cube family. Polyhedron cubes, spherical cubes, tetrahedron cubes, mirror cubes, gear cubes, cake cubes and other particularly shaped cubes have a wide range of structural shapes. Figure 4 lists six cases of specially shaped cubes.

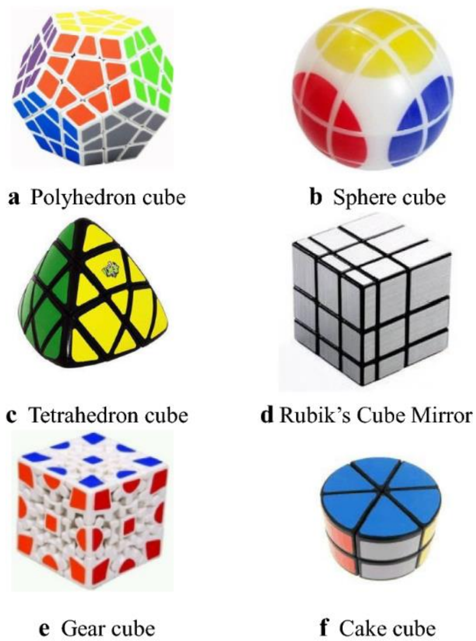


Figure 4: Models of Specifically Shaped Rubik's Cube

3.0 Mathematics Theorem about Rubik's Cube

3.1 Permutations

3.1.1 By Turning The Corner Cubes and Edges of The Rubik's Cube

The initial Rubik's Cube, which is 3×3×3, had eight corners and twelve edges.

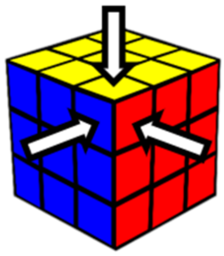
The Eight Corner Cubes

To compose and arrange the eight corner cubes, there are 40, 320 methods. The calculation is as below:

$$8! = 40{,}320$$

(8 represents the eight corner cubes)

The figure below shows that each corner cube has three possible orientations.



Of the eight corner cubes, seven of them can be oriented independently and the eighth corner cube depends on the other seven corner cubes, giving 2,187 combinations. The calculation is as below:

$$3^7 = 2{,}187$$

(3 represents the three orientations for each corner cube)

(7 represents the seven independent oriented corner cubes)

The Twelve Edges

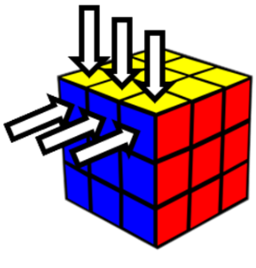
To arrange the twelve edges, there are 239, 500, 800 methods. The calculation is as below:

$$\frac{12!}{2} = 239{,}500{,}800$$

(12 represents the twelve edges)

(An even permutation of the edges implies even permutation of the cornered elements.)

The figure below shows that each edge has two possible orientations.



Of the twelve edges, eleven of them can be flipped independently and the twelfth edge depending on the other eleven edges, giving 2, 048 combinations. The calculation is as below:

$2^{11} = 2{,}048$
(2 represents the two orientations for each edge)
(11 represents the eleven independent flipped edges)

Total Combinations

The number of combinations which can be formulated by a Rubik's Cube is 43,252,003,274,489,856,000. The calculation is as below:

$$8! \times 3^7 \times \frac{12!}{2} \times 2^{11} = 43{,}252{,}003{,}274{,}489{,}856{,}000$$

From this calculation, the resolution of the Rubik's Cube is approximately 43 quintillion.

However, the total combinations demonstrated above is limited to the number of permutations which can be achieved by turning the edges of the cubes.

3.1.2 By Disassembling the Rubik's Cube

If an individual considers the potential permutations which can be reached through disassembly of the Rubik's Cube, the total combinations become multiplied by a factor twelve, which is 519,024,039,293,878,272,000. The calculation is as below:

$$8! \times 3^7 \times \frac{12!}{2} \times 2^{11} \times 12$$
$$= 8! \times 3^7 \times \frac{12!}{2} \times 2^{11} \times (3 \times 2 \times 2)$$
$$= 8! \times 3^8 \times 12! \times 2^{12}$$
$$= 519{,}024{,}039{,}293{,}878{,}272{,}000$$

From this calculation, the potential configurations of the Rubik's Cube are approximately 519 quintillion but only one in twelve of these are solvable. This is because of the nature of the ordering of motions which will swap a single pair of pieces or twist solitary corners of the cube edge. Therefore, there are 12 possible sets of reachable configurations, which are also designated as universes or orbits into which the cube can be placed by deconstructing and reconstructing it.

**Remark: The preceding total combinations assume the centre faces are in a fixed position.

3.2 Central Faces

The original Rubik's Cube had no orientation markings on the centre faces (although some carried the words "Rubik's Cube" on the centre square of the white face), and therefore solving it does not require any attention to orienting those faces correctly. However, Markers could be drawn on the central cubies of the Rubik's cube which is not in a changed and resolved state in a manner where the drawings match each of the central cubies. The cube that is marked in this way is referred to as a "super cube" **(Refer to Image 3.2.1)**. The drawing on the super cubes raises the level of difficulty due to the nature of increasing the collection of different potential set ups.

Image 3.2.1 Super Cube

There are over 200 ways of directing the central parts of the cubes, due to the quality that a corner permutation has the meaning of an even number of rotations of the central sides. Apart from the directions of the core faces, the cube can be decrypted. The number of central squares that require a quarter twist will always be an even amount. The possible number of permutations of the three-plane Rubik's cube grows as a result of the direction of the central sides from 4.3 x 1,019 to 8.9 x 1,024. A Rubik's cube's twisting causes a change in the number of permutations, which is brought on by the alteration in the number of central faces. In general, there are 6! methods to arrange the cube's central six sides. Without redoing the cube, 24 of these permutations are feasible.

3.3 Algorithms

Algorithm is a list of well-defined instructions for solving the cube which can be used to bring the cube closer to being solved. Many algorithms are made to alter only a small part of the cube without interfering with other parts that have previously been solved in order to be applied repeatedly to various portions of the cube until the entire cube is solved.

The Fridrich method (CFOP method) is a fast method algorithm for solving the Rubik's Cube created by Jessica Fridrich. The Fridrich method consists of 4 steps: Cross, First Two Layers (F2L), Orient Last Layer (OLL), and Permute Last Layer (PLL). It's one of the fastest speedcubing methods which requires us to memorise up to 78 different algorithms.

i) Cross

Cross is the first step of the Fridrich method for solving the Rubik's cube. In this step, 4 edges have to be solved to their correct positions. The most often chosen side to make a cross on, by convention, is white. Most people are thus said to make a 'white cross'.

ii) First Two Layers (F2L)

F2L is the longest step in the CFOP method and one of the most important to work on if someone wants to get much faster because the cube is solved layer by layer. There are effectively 41 F2L cases that have 41 algorithms if each corner-edge position is counted.

Besides that, Keyhole is an effective method for solving certain F2L cases. It is deals with the F2L part of the CFOP method. We can think of it as an in-between method between Layer-By-Layer and Full F2L. The standard procedure involves cutting off the corner, connecting it to its edge, and putting it into the slot. With a keyhole, it is possible to shift the D layer (Refer to **Image 4.1 Rubik's cube notation**) away and replace it with an empty slot. The pair is then solved by swiftly inserting the edge and repositioning the D layer. A solved edge and an unsolved corner can likewise be combined using this technique.

Multislotting is an extension to F2L that solves two corner-edge pairs at once, or more accurately, modifies the insertion for the first pair to set up the second.

iii) Orient Last Layer (OLL)

The first class of algorithms is called Orient Last Layer (OLL), which solves the top face of the last layer. There are 57 different algorithms to solve each of the 57 possible patterns.

iv) Permute Last Layer (PLL)

The second class of algorithms is called Permute Last Layer (PLL), which solves the rim of the last layer (and, as a result, the entire cube). PLL, the acronym for Permutation of the Last Layer, is the last step of many speedsolving methods (Refer to **4.3.8 Permutations of last layer (PLL)**). In this step, the pieces on the top layer have already been oriented (OLL) so that the top face has all the same colour, and they can now be moved into their solved positions. There are 21 PLL algorithms.

3.4 Relevance and Application of Mathematical Group Theories

The Rubik's Cube is made for the use of mathematical group theory, which has been helpful for the reasoning of specific algorithms.

3.4.1 Structure in Algorithms

These algorithms have a commutator structure which is shown by $XYX^{-1}Y^{-1}$. X and Y are special techniques such as specific moves or move-sequences while X^{-1} and Y^{-1} are inverses of these techniques (X and Y).

A conjugate structure can also be used which is shown by XYX^{-1}. This structure is normally referenced by speed cubers as beginning moves, in other words, a "setup move".

3.4.2 Subgroups

Besides, the characteristic of having well-defined sub-groups within the Rubik's Cube group enables the puzzle easier to be learned and mastered by moving up through various difficult levels. For example, one of the difficult levels could involve solving cubes that have been coded by the application of twists using only 180-degree turns.

The scrambling of the Rubik's Cube is an important quality. This scrambling means that the setting of the Rubik's Cube would require the maximum amount of twist to return the cube to its original condition. This has been calculated to be a maximum value of 22 in 2009 and has been shown to be God's number.

The lower limit on God's number is defined. The first turns of the sides of the Rubik's Cube can take place in a dozen of different methods as there are six sides which can be turned in two potential directions and the motion that affect the turn can take place in an additional 11 set ups, the limits on the maximum number of moves which are different from the original condition can be shown by the mathematical relationship of:

$12 \times 11^{n-1} \geq 4.3252 \times 10^{19}$. This equation is resolved by $n \geq 19$.
(12 represents the twelve edges)
(11 represents the additional eleven set ups)
(4.3252×10^{19} represents the resolution of the Rubik's Cube where the centre faces are fixed)

These subgroups are the principle underlying the computer cubing methods by Thistlethwaite and Kociemba, which solve the cube by further reducing it to another subgroup.

3.4.3 Thistlethwaite's Algorithm

Thistlethwaite's algorithm works by restricting the positions of the cubes into groups of cube positions that can be solved using a certain set of moves. The cube is solved by moving from group to group, using only moves in the current group. The groups are as below:

$$G_0 = \langle L, R, F, B, U, D \rangle$$

This group contains all possible positions of the Rubik's cube.

$$G_1 = \langle L, R, F, B, U^2, D^2 \rangle$$

This group contains all positions that can be reached from the solved state with quarter turns of the left, right, front, and back sides of the Rubik's Cube, but only double turns of the up and down sides.

$$G_2 = \langle L, R, F^2, B^2, U^2, D^2 \rangle$$

This group contains all positions that can be reached with only double turns of the front, back, up, and down sides and quarter turns of the left and right sides.

$$G_3 = \langle L^2, R^2, F^2, B^2, U^2, D^2 \rangle$$

This group contains all positions that can be solved using only double turns on all sides.

$$G_4 = \{1\}$$

The final group contains only one position, the solved state of the cube.

<u>Example of The Application of Thistlethwaite Algorithm</u>

A scrambled cube always lies in G_0. A look up table of possible permutations is used that uses quarter turns of all sides to get the cube into group G_1. Once in G_1, quarter turns of the up and down sides are not allowed in the sequences of the lookup tables, and the tables are used to get to group G_2. Once in G_2, quarter turns of the front, back, up, and down are not allowed in the sequences of the lookup tables, and the tables are used to get to group G_3. Once in G_3, quarter turns of all sides are not allowed to get the cube into group G_4. Once in G_4, the Rubik's Cube is in the solved state.

3.4.4 Kociemba's Algorithm

Kociemba's algorithm is the improvement of Thistlethwaite's algorithm by Herbert Kociemba in 1992. Herbert Kociemba reduced the number of intermediate groups to only two. The groups are as below:

$$G_0 = \langle U, D, L, R, F, B \rangle$$

This group contains all possible positions of the Rubik's cube.

$$G_1 = \langle U, D, L^2, R^2, F^2, B^2 \rangle$$

This group contains all positions that can be reached with only double turns of the left, right, front, and back sides and quarter turns of the up and down sides.

$$G_2 = \{1\}$$

The final group contains only one position, the solved state of the cube.

4.0 Solutions of The Rubik's Cube

There are several ways to solve the Rubik's cube. Here, we will explain one of the most popular ways to solve it.

4.1 Notations

Before learning how to solve the Rubik's cube, we need to learn some notations to turn the Rubik's cube.

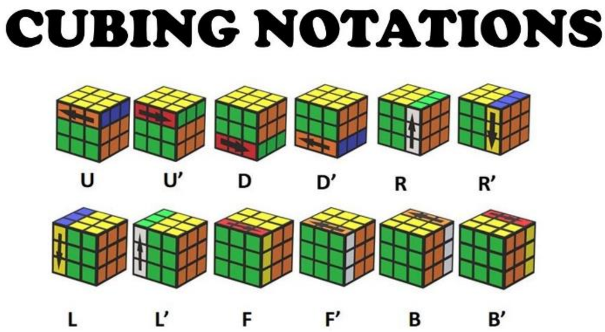


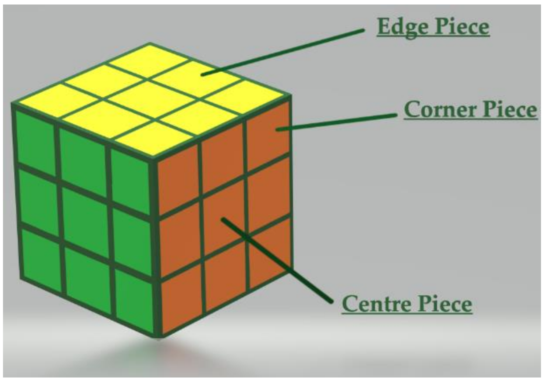
Image 4.1 Rubik's cube notation

R: Rotate the right layer clockwise.
R': Rotate the right layer anti-clockwise.
L: Rotate the left layer clockwise.
L': Rotate the left layer anti-clockwise.
U: Rotate the top layer clockwise.
U': Rotate the top layer anti-clockwise.
F: Rotate the front layer clockwise.
F': Rotate the front layer anti-clockwise.

*The symbol, prime ('), is considered the opposite turn of the original direction.

4.2 Concepts

There are 12 edge pieces and 8 corner pieces. We can move the edge pieces and corner pieces by simply tuning any side of the cube. The centre pieces will always stay at its original position however you turn it.



For a standard Rubik's cube, the opposite colour for each colour is fixed.

The opposite of white is yellow.

The opposite of blue is green.

The opposite of red is orange.

Image 4.2 Centre, edge and corner pieces

4.3 Steps to Solve The Rubik's Cube

4.3.1 <u>Solving the cross</u>

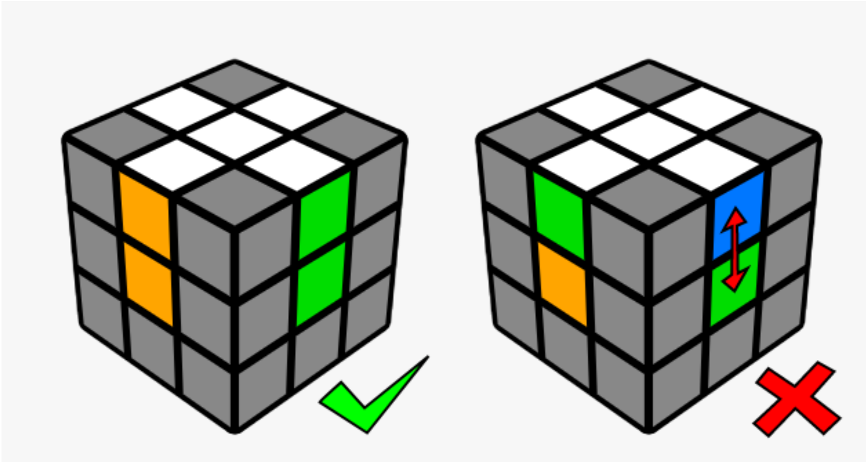
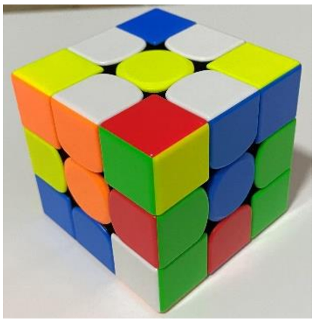


Image 4.3.1.1

**The edges pieces of the cross must be aligned to the lateral sides of the same colour.

 **Image 4.3.1.2: A unaligned white cross**	**Step 1:** We start by choosing any centrepiece to do our cross. In this case, let it be white. 4 white edge pieces need to be brought adjacent to the white centrepiece. (Refer to **Image 4.3.1.2**)
 **Image 4.3.1.3: One aligned white edge cross to its corresponding side colour**	**Step 2:** We align one of the white edge pieces to its corresponding centre colour piece by matching the side colour of the white edge piece to the same-coloured centrepiece. (Refer to **Image 4.3.1.3**)

 **Image 4.3.1.4 A daisy**	**Step 3:** Then we do 2 **R** moves to preserve the edge piece, which in case **Image 4.3.1.3**, we turn the orange side 2 times. Then, we repeat **Step 2** for the other 3 edge pieces. Then we will get, what we say, a daisy pattern. (Refer to **Image 4.3.1.4**)
 **Image 4.3.1.5: Aligned and correct cross**	**Step 4**: We now do 2 **R** moves on every side of the white centrepiece and we will get a correct white centre cross. (Refer to **Image 4.3.1.5**)

4.3.2 Solving the first layer

Before solving the first layer, we need to memorise an algorithm called the ‘sexy’ move. The notation for it is **R U R’ U’**.

 **Image 4.3.2.1: A corner piece aligned vertically to its original place**	**Step 1:** We first hold the cube white side facing down. The upper centrepiece will be yellow. Then, we find a corner piece with a white side. Let's say a white-blue-orange corner piece. Do **U** moves until the corner reaches the correct side of the cube, which in this case, is between the blue and orange side. (Refer to **Image 4.3.2.1**)
 **Image 4.3.2.2: A correctly placed corner piece**	**Step 2**: We hold the cube white facing down and the corner piece from **Step 1** on our right hand side. Then, we do the ‘sexy” move algorithm until the corner piece is correctly placed. (Refer **Image 4.3.2.2**)
 **Image 4.3.2.3: A solved first layer**	**Step 3:** We repeat **Step 1** and **Step 2** for the other 3 white corner pieces. Then we will get a correctly finished first layer. (Refer **Image 4.3.2.3**)
 **Image 4.3.2.4: An unsolved white corner on the white face**	**Step 4**: If there is no more white corner piece on the yellow side, we find an unsolved corner piece from the white side (**Image 4.3.2.4**). Then we do a ‘sexy’ move on the corner. Then continue to **Step 1** and **Step 2.**

4.3.3 Solving the second layer

To solve the second layer, we must learn the left side 'sexy move' with notation **L' U' L U.**

Step 1:
We find an edge piece which is not the colour opposite of our cross (the yellow edge piece). In our case, we start with the green-orange edge. We align the edge piece to its corresponding face by matching the colour facing away from the yellow face.

After **Step 1**, we will get 2 types of cases,
(a) the edge piece needs to be inserted to the right (Refer to **Image 4.3.3.1a**), and
(b) the edge piece needs to be inserted to the left (Refer to **Image 4.3.3.1b**).

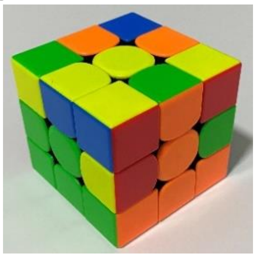


Image 4.3.3.1a

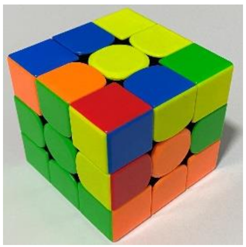


Image 4.3.3.2b

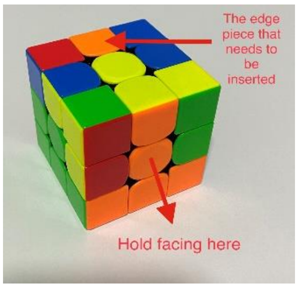


Image 4.3.3.3

Image 4.3.3.4 A solved edge piece

Step 2(a):
For case (a), we first do a **U** move. Then, a right 'sexy' move which is **R U R' U'**.
Then we turn the whole cube so that the edge that needs to be inserted is opposite from us. (Refer to **Image 4.3.3.3**) We then do a left 'sexy' move which is **L' U' L U**. The edge piece will then be inserted. (Refer to **Image 4.3.3.4**)

Step 2(b):
For case (b), we will just do an inverse of **Step 2(a)**. We first do a **U'**, then a left 'sexy" move. Turn the whole cube so that the edge that needs to be inserted is opposite from us and then a right 'sexy' move.

Image 4.3.3.5 First two layers solved

Step 3:
We repeat **Step 1** and **Step 2** for the other 3 edge pieces and the second layer will be solved. (Refer to **Image 4.3.3.5)**

4.3.4 Making the last layer cross

There will be 3 cases,
(a) the dot (Refer to **Image 4.3.4.1**),
(b) the 'L' shape (Refer to **Image 4.3.4.2**), and
(c) the line (Refer **Image 4.3.4.3**)

Image 4.3.4.1 The dot

Image 4.3.4.2 The 'L' shape

Image 4.3.4.3 The line

Image 4.3.4.4

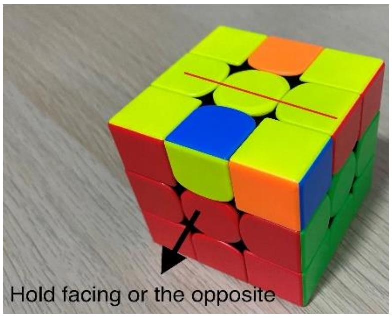


Image 4.3.4.5

If we get (a),
We can do an algorithm which is **F ('sexy' move) F'** or **F R U R' U' F'** to get (b).

When we get (b),
We can repeat the algorithm but the 'L' must be located as shown in **Image 4.3.4.4**. The 2 edge pieces must be located at the left and at the back of where you will perform the algorithm. Then, you will get (c).

When we get (c),
We repeat once more the algorithm but the line must be holded horizontally as shown in **Image 4.3.4.5.**

Image 4.3.4.6 The last layer cross

Therefore, we will get a cross on the last layer. (Refer to **Image 4.3.4.6**)

4.3.5 Make a full coloured face

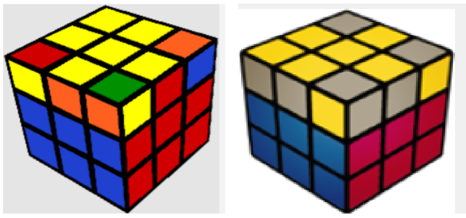


Image 4.3.5.1: The sune and anti-sune

Image 4.3.5.2: A solved last layer face

Keep performing the algorithm below until you get the sune or anti-sune (Refer to **Image 4.3.5.1**):
R U R' U R U2 R'

After getting either one of these situations, use the algorithm below to solve the last layer face.
The sune: **R U R' U R U2 R'**
Anti-sune: **R U2 R' U' R U' R'**

Image 4.3.5.3: All the last layer cross algorithms
(Source: https://www.cubeskills.com/uploads/pdf/tutorials/oll-algorithms.pdf)

For a faster method on solving the last layer cross, we can memorise more algorithms. The list shows all the possible outcomes of how a last layer cross can be arranged.

4.3.6 Solve the last layer corners

There will be 2 cases,
(a) a side with correct corners or what we call headlights (Refer to **Image 4.3.6.1**) and (b) no headlights (Refer to **Image 4.3.6.2**)

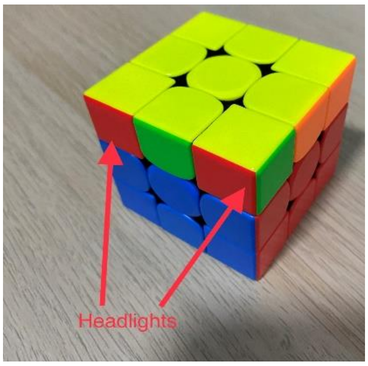


Image 4.3.6.1
A side with headlights

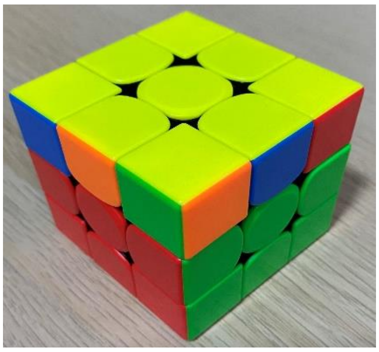


Image 4.3.6.2:
4 sides without a pair of headlights

Image 4.3.6.3 A fully solved cube except for the last edge pieces

For (a),
We perform this algorithm below with the headlights on your left.
R U R' U' R' F R R U' R' U' R U R' F'

For (b),
We perform this algorithm on any side.
F R U' R' U' R U R' F' R U R' U' R' F R F'

4.3.7 Solving the last layer edges

There will be 2 cases,
(a) edge pieces need to be solved clockwise (Refer to **Image 4.3.7.1**) and
(b) edge pieces need to be solved counter-clockwise (Refer to **Image 4.3.7.2**)

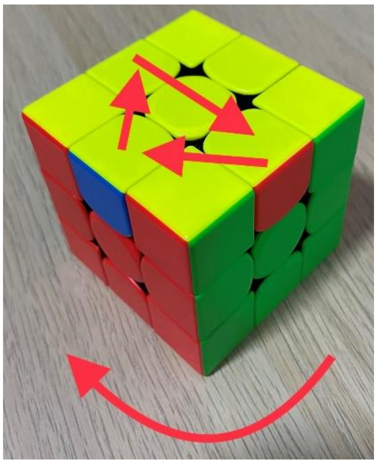


Image 4.3.7.1

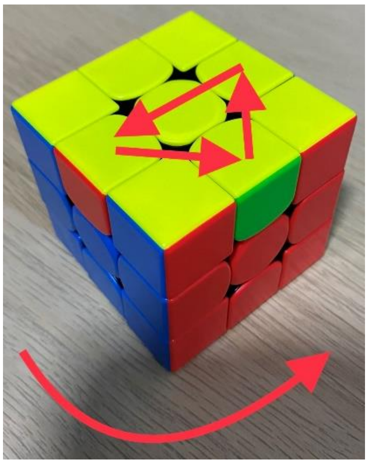


Image 4.3.7.2

For (a),
We perform this algorithm below on the only fully solved face of the 4 last layer sides.
R' U R' U' R' U' R' U R U R R

For (b),
We perform this algorithm below also on the only fully solved face of the 4 last layer sides.
R R U' R' U' R U R U R U' R

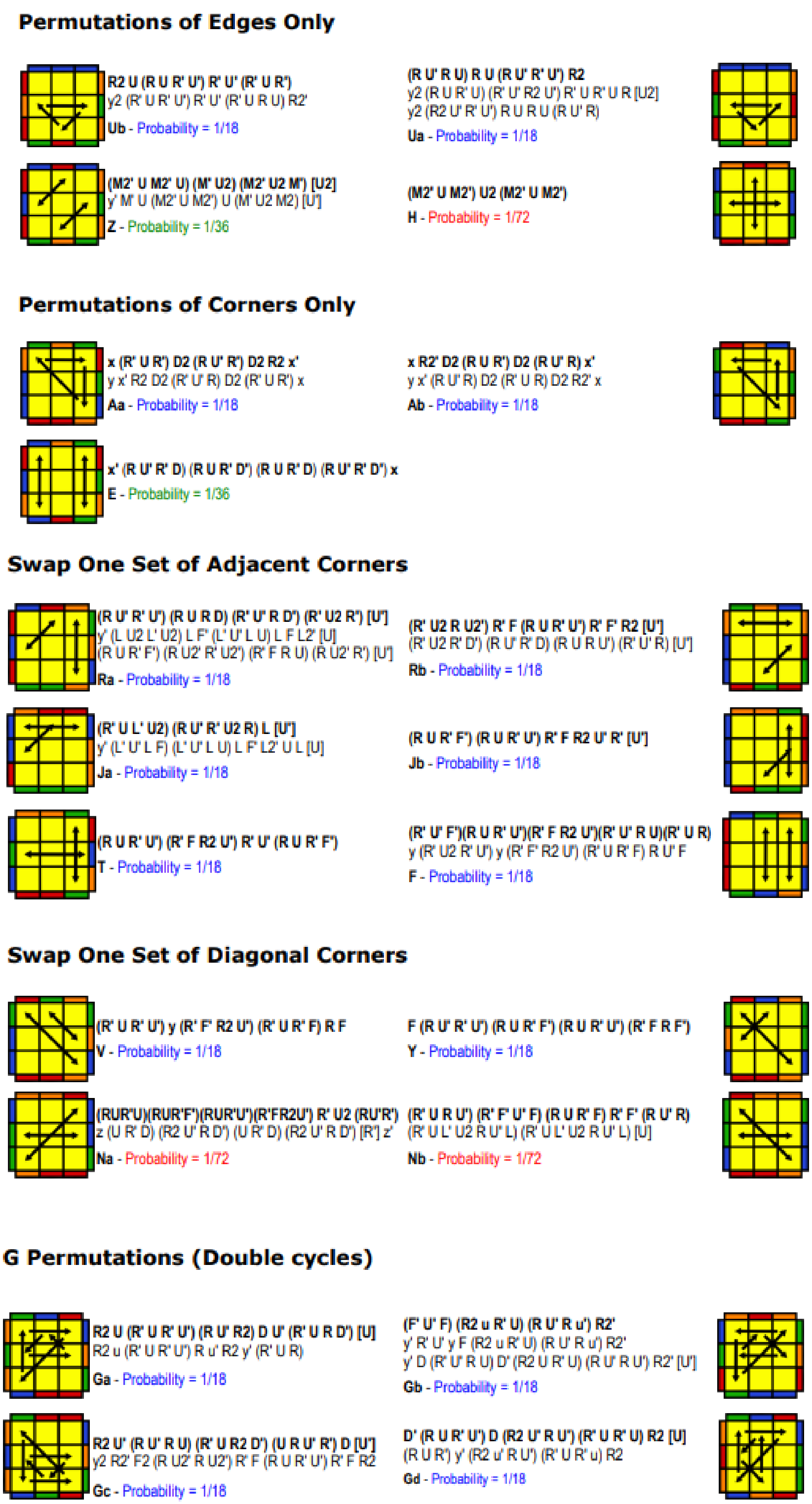
If you didn't get any of the 2 cases, you can do either the (a) or (b) algorithm once and you will get case (a) or (b). Then, just follow the steps above.

Hence, the Rubik's cube is completely solved.

4.3.8 Permutations of last layer (PLL)

As it takes time to solve the steps mentioned in **Section 4.3.6** and **Section 4.3.7**, there's an alternative to solving the last edges and corners of the Rubik's cube instantly with only one algorithm.

The algorithm below shows how to solve the last layer Rubik's cube in one step with the possible situations.



Conclusion

There are numerous rubik cube variations and also there are many ways to solve it. We discovered several ways of solving the rubik's cube. Two theorems were studied in this report namely permutation and algorithm. We get to know more about the mathematics theorem in order to solve the rubik's cube. From the mathematics theorem studied, we managed to apply that on solving rubik's cube. This eventually helps us to develop thinking abilities and problem-solving skills.

One of the most popular ways of solving the rubik's cube is by cubing notations. Fixing a notation will ease our work. Firstly, the edges of the cross must be aligned to the lateral sides of the same colour. Then the first layer should be solved by using the algorithm called the ‘sexy’ move. Followed by the second layer using the same algorithm. Align the last layer's corner and then make a full coloured face. Next, solve the last layer corners. Followed by the last layer edges. Then the rubik cube is solved.

The external characteristics of Rubik’s Cube have been studied and applied in multidisciplinary fields, so the principles of the internal structure of Rubik’s Cube should be explored at the same time.

The relationship between Rubik's Cube and mechanism was presented in this study, and research into Rubik's Cube in the field of mechanism is still in its early phases. The new Rubik's Cube mechanism difficulties should be investigated, and a systematic theory of the Rubik's Cube mechanism should be developed. Some of the research findings will be used to guide the development of the Rubik's Cube mechanism in mechanical engineering applications. It is important to look into the cube mechanism and foster the development of the cube structure. The topology theory of the cube mechanism has yet to be investigated further.

Personal Reflection

CH'NG JIA QI U2103224

Before the community project, I was amazed with the mathematics' theorem behind the Rubik's Cube. During the process of researching, I found out that it's not easy to filter out the information that is suitable to the community, so we spent most of our time simplifying the information we got, so that we could use the simplest words to present it.
On the day of the presentation, I feel so happy when the community gets what we are trying to deliver and gives good feedback from the interaction session.
Overall, I think the cooperation between the community partners is the most important quality in group work. I am grateful to have my partners because all of us complete our parts on time and smoothly, making our project perfect.

CHAU PUI KEI U2103460

This is a completely new experience from researching, preparing until running the community project. Before the day of the community project, we prepared the slides for sharing but we faced some problems such as the place for sharing is not available. Fortunately, with the cooperation of me and my community partners, we were able to solve the problems in a short time.
I have never thought that a Rubik's Cube can be so interesting and can be shared among a community! I'm surprised that many people are interested in the Rubik's Cube while the community project is being carried out.
From this community project, I learn the importance of teamwork and the ability of solving a problem in a limited time. I found out that adaptation is very important.

GAVIN LIM ZHEN YAO U2103511

It was a wonderful experience for me. Before participating in the community event, I never felt that Mathematics can be a medium for us to bond with people. Since I ever learned to solve the Rubik's Cube myself, I would only discuss it only with my friends who also knew how to solve the Rubik's Cube.
During the preparation of this project, I found out that there are also ways for me to communicate with those who have little to zero knowledge of the Rubik's Cube.
From this community project, I found out that just by sharing our knowledge of Mathematics and for this community project, the Rubik's Cube, we can reach out to all types of people in the community.

KANCHANAA A/P SIVABALAN U2103645

I was excited from the start till the end of the community event. Throughout this project I have experienced both good and unexpected moments and I have come to realise that teamwork and good communication among the members are important so that we can tackle one problem as soon as possible.
I was glad that we conducted this community project about Rubik's cube since we were able to share the information and solutions for the world's famous puzzle to many people. Besides, I was able to learn extra new ways to solve the Rubik's cube in a shorter period.

Overall, it was a new exposure for me as I was able to interact with the people and realised that mathematics profoundly changed my view of the world with new eyes.

KUNG SOOK KEAN U2103458

The process of organizing a community event about the Rubik's Cube provided me with a valuable chance to not just learn about the historical background of the Rubik's Cube but to develop a unique contribution to this topic. Our project began with the board's desire to learn more about how to solve it and try to understand its theorem. I am so impressed that there are so many theorems of the Rubik's Cube, such as F2L and crossing, as I am the person in charge of the mathematical theorem of the Magic Cube. The research for this project involved a long process of elimination and rejection, as typical for any project. We must investigate and discuss how to imply the theorem in solving the Rubik's Cube. Using a mathematical theorem is significant for solving everything in life, not only studying.

MEENA A/P MURUGAPPAN ANNAMALAI U2103652

I had a very great experience organising such community events. Solving Rubik's cube was never a simple one for me but when we decided to teach them to the community, I was excited. I took effort to learn all the background of Rubik's cube and also the way to solve it. Discovering Rubik's cube gave me the knowledge that there are many mathematical theorems involved in it. Although it took some time to understand the theories behind Rubik's cube , the process of learning it was fun and very knowledgeable. Moreover, teaching the community how to solve it was such a great experience. In conclusion, I discovered the mathematical theorem behind the solving process of the Rubik's cube and gained experience in teaching the community.

NG JIA XUAN U2103227

It was a successful community project for me also I really enjoyed myself throughout the process of organising this community project. From the project, I had an opportunity to know more about Rubik's cube. Through this community project about Rubik's cube, I discovered a few mathematical theorems that used to solve the Rubik's cube also its history background as well. On the presentation day, I'm grateful that our group was able to deliver the right information about Rubik's cube to the community and luckily, we received good feedback. Besides, cooperation and teamwork between groupmate is the essential part that leads this project towards success. In sum, it is a new experience for me to interact with the community and I appreciate that mathematics is not only studying but solving our life's problems.

TAI KAH YIT U2103464

Before the community project, I had no experience in solving Rubik's cube. By organising this community event, I did some research on the history and background of the Rubik's cube. I found that there are many types of Rubik's cubes instead of only 3x3x3 cubes. I also learnt to solve the Rubik's cube layer by layer and use the mathematical theorem.
Besides, I also learnt how to design presentation slides by using canvas. We distributed our tasks evenly, and every member worked together to make this project successful. Teamwork is the key to making this project successful.

In conclusion, this community project taught me hard and soft skills.

TENG JIAN HONG S2132490

This is a unique experience for me. Before the community project, I always thought Rubik's cube is just a simple puzzle game that relies on algorithms. But after the project, I found out that Rubik's cube has so many maths theorems involved. When I first learn how to solve a Rubik's cube 6 years ago, I always think that there's only one way to solve it, but after the project, I found out that there are many ways to solve a Rubik's cube and it's very different from the way that I've learnt.

While doing this project, I've also learnt how to make the slides for solving a Rubik's cube for people who have no knowledge about solving one.

From this community project, I've learnt the importance of mathematics and it's widespread uses not just from academics, but also for games like Rubik's cube.

References

Alexandra Alter. (2020, September 16). *He Invented the Rubik's Cube. He's Still Learning From It.* The New York Times. https://www.nytimes.com/2020/09/16/books/erno-rubik- rubiks-cube-inventor-cubed.html

Deutsch, M. (2017, January 9). *M2M Day 69: Decoding Rubik's Cube algorithms*. Medium. https://medium.com/@maxdeutsch/m2m-day-69-decoding-rubiks-cube-algorithms-6ea7e7704ec9

F2L Algorithms Page. Rubiksplace.com. http://www.rubiksplace.com/speedcubing/F2L-algorithms/

Feliks, Z., & Andy, K. *OLL Algorithms (Orientation of Last Layer).* https://www.cubeskills.com/uploads/pdf/tutorials/oll-algorithms.pdf

Feliks, Z., & Andy, K. *PLL Algorithms (Permutation of Last Layer).* https://www.cubeskills.com/uploads/pdf/tutorials/pll-algorithms.pdf

Gamesver Team. *History of The Rubik's Cube (Timeline): 20 Important Moments (From Inception…)*. gamesver. https://www.gamesver.com/history-of-the-rubiks-cube-timeline-important-moments-from-inception/

Important Dates in Rubik's Cube History. Ruwix. https://ruwix.com/the-rubiks-cube/history-rubiks-cube/important-dates-timeline/

Jaap's Puzzle Page.Thistlethwaite's 52-Move Algorithm. https://www.jaapsch.net/puzzles/thistle.htm.

Pranav Prabhu. (2022, February 21). *3 Useful Advanced f2L Skills/Techniques For Speedsolving.* cubelelo. https://www.cubelelo.com/blogs/cubing/useful-advanced-f2l-techniques-for-speedsolving

Rubik's Cube. (2014). The Strong National Museum of Play. https://www.museumofplay.org/toys/rubiks-cube/

The Mathematics behind the Rubik's Cube, Math Problem Example. Essays.io. https://essays.io/the-mathematics-behind-the-rubiks-cube-math-problem-example/

Two-Phase Algorithm Details. http://www.kociemba.org/math/imptwophase.htm.