The Mega Distributed Lag Model

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The Mega Distributed Lag Model

Mario Arturo Ruiz Estrada[1], Evangelos Koutronas[2], Ross Knippenberg[3]

ABSTRACT This paper attempts to describe the graphical behavior of the distributed lag model in an infinite coordinate space. The "mega distributed lag model" (MDL) is a mathematical framework that can examine the simultaneous interrelationships between all involved variables. The multidimensional graphical setting simultaneously reveals all non-linear exposure—response dependencies and delayed effects between lagged and dependent variables—which two-dimensional figures overwhelmingly fail to capture. Under the Omnia Mobilis assumption, each distribution lag function is indexed with respect to time and space. The Mega distributed lag model observes multiple trends in full motion, the final output (determinant) of which is called "the JIM-coefficient". Hence, this paper tries to analyze different approaches of lag distribution models that can help in the construction of our new model. The mega distributed lag model" (MDL) is moving from the uses of the classic 2-dimensional and 3-dimensional graphical modeling to a multidimensional graphical modeling in Econometrics. Finally, this model is an extension of those explored earlier in the field of econographicology.

KEY WORDS: Econographicology, Distributed lag model, Multi-Dimensional graphs and Multi-Dimensional geometry

JEL Classification: C10

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1. Introduction

Dynamic specification attempts to capture the hidden dynamics derived by the observed time-series data in a postulate theoretical model (Hendry, Pagan, & Sargan, 1984). The general approach in a trend stationary process has been to de-trend the series and to model the de-trended series as distributed lag or autoregressive distributed lag (ADL) models. A lag distribution function gives a magnitude of the coefficient of a lagged explanatory variable, expressed as a function of the lag, and extra information is injected into the estimation procedure. Estimation and inference concerning the long-run properties of the model are then carried out using standard asymptotic normal theory. A wide variety of specifications has been suggested for this purpose, including arithmetic, inverted V, Almon, Shiller, harmonic geometric, Pascal, rational, gamma, and exponential (Kennedy, 1998). A distributed-lag model is a dynamic model in which the effect of a regressor x on y occurs over time rather than all at once. Based on a Hendry-style (Hendry et al., 1984) autoregressive distributed lag equation (see Appendix A), considered the general form of one dependent variable and one explanatory variable, the model takes the form

$$y_t = \alpha + \beta(L)\, x_t + u_t = \alpha + \sum_{s=0}^{\infty} \beta_s\, x_{t-s} + u_t \qquad (1)$$

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where x_t is exogenous for t= 1,…,T, and u_t is the stationary error term, i.i.d. $(0, \sigma^2)$. This form is very similar to the infinite-moving-average representation of an ARMA process, except that the lag polynomial on the right-hand side is applied to the explanatory variable x rather than to a white-noise process ε. The individual coefficients β_s are called lag weights and collectively comprise the lag distribution. They define the pattern of how x affects y over time.

The distributed lag concept was known among economists through the pioneering work of Irving Fisher (1930) on the relationship between the nominal interest rate and the expected inflation rate. However, its application in econometrics was launched by Koyck's (1954) geometric distributed lag model in a study of investment. The geometric distributed lag model was based on the notion of adaptive expectations hypothesis. It could estimate the autoregressive model with an auto-correlated error. One disadvantage of this lag structure is that coefficients of the lagged explanatory variables continually decline; they cannot first rise and then decline, a pattern thought by many to be a priori attractive and one that should not be ruled out of consideration. One way of addressing this problem is to allow unrestricted coefficients on the first few lagged variables and then impose a geometric pattern (Kennedy, 1998). The above mechanism was extensively applied in numerous empirical studies: Cagan's (1956) study of monetary aspects of hyperinflation; Friedman's (1957) study of consumption behavior; Nerlove's (1958a) study of supply and demand decisions (cobweb model); Solow's (1960) study of consumption behavior; and Jorgenson's (1966) study of investment.

At about the same time, Almon (1965) provided a polynomial generalization of Fisher's (1937) arithmetic lag distribution. In this technique, the n coefficients of the lagged explanatory variables are assumed to lie on a polynomial of order r. This allows for a flexible lag structure reduction in the number of the parameters that require estimation if r+1 is less than n. It can be viewed as imposing a specific set of linear constraints on OLS estimation. Several studies have investigated the effects of misspecifying the lag length or polynomial degree in the Almon lag model (Frost, 1975; Harper, 1977; Schmidt & Sickles, 1975;

Schmidt & Waud, 1973; Terasvirta, 1976; Thomas, 1977; Trivedi, 1970; Trivedi & Pagan, 1979).

Almon polynomial lag distribution was later extended further by Shiller (1973). Shiller's distributed lag is a variant of this in which these restrictions are stochastic, incorporated via the mixed estimation technique; the coefficients of the lagged explanatory variable lie close to, rather than on, a polynomial. The main problem with the Almon lag is determining n and r; pretesting is usually employed for this purpose, resulting in estimators with unknown properties.

Mitchel and Speaker (1986) introduced a reduced form of the model via a linear transformation of the polynomial lag model, estimated the reduced-form parameters, and recovered estimates of the structural parameters via an inverse linear transformation of the reduced-form parameter estimates. The so-called polynomial inverse lag model has flexible shape and can be easily estimated linearly as long as the lag operator is invertible (the polynomial roots inside/outside the unit root circle) (Kasparis, 2008). Other forms of dynamic specification considered in the literature included the partial adjustment model (Eisner & Strotz, 1963; Nerlove, 1958b) and the multivariate flexible accelerator model (Treadway, 1971) and Sargan's (1964) work on econometric time series.

The most known parameterization of the (ADL) model is the error-correction model (EC). The use of an error correction mechanism for non-stationary variables is equivalent of performing co-integration (Dickey & Fuller, 1981). By differencing and forming a linear combination of the non-stationary data, all variables are transformed equivalently into an EC model with stationary series only (Hassler & Wolters, 2006). Phillips (1954; 1957) introduced EC models to economics. Sargan (1964) used them to estimate structural equations with auto correlated residuals, and Hendry popularized their use in econometrics in a series of papers. A survey on specification, estimation and testing of EC models is given by Alogoskoufis and Smith (1995).

Current literature, however, has identified three major shortcomings (Kim, Fraser, & Hyndman, 2010). First, point estimates of the long-run response are often extremely disparate (Askari & Cummings, 1977; Marquez & McNeilly, 1988). Second, the sampling variability of the long-run response estimator is

difficult to estimate accurately (Li & Maddala, 1999). Third, the long-run response estimator can be severely biased, especially in small samples (Focareli, 2005; Pesaran & Zhao, 1999). Among the alternative distributed lag models is a particularly interesting paper published by Grace Wahba (1969), with a chapter entitled "The estimation of the coefficients in a multidimensional distributed lag model". In it, she proposes a multi-dimensional theoretical framework scheme for the general distributed lag model. It is based on the use of matrices and vectors to trace the multi-dimensional effects of the various lags, so the idea of a multi-dimensional focus for the distributed lag model is nothing new. However, the multi-dimensional distributed lag model proposed by Wahba lacks a multi-dimensional visualization to illustrate the behavior of which it speaks. Instead, Wahba's paper utilizes the same 2-dimensional graphical model as its predecessors, and, in doing so, fails to incorporate the dynamic behavior of multidimensional variables as a whole.

2. The Mega Distributed Lag Model

The mega distributed lag model function is formed by a combination of variables, each classified into four levels. Each classification level may contain infinitely–many variables. From largest to smallest, these classifications are: general-variables (GV), sub-variables (SV), micro-variables (MV) and JI-variables (JIV) (see expression 2.5). Each classification varies constantly at different rates. From a graphical perspective, the drawback of a typical distributed lag model is that a researcher can visualize it only in a two-dimensional coordinate space. While this gives a simple representation of the relationship between two variables, the visualization is too restricted to be of use. Namely, the two-dimensional representation ignores the simultaneous interrelationships between all variables, which could adversely affect the analysis. Visualization of the mega distributed lag model function requires the use of a multi-dimensional coordinate space. The most suitable multi-dimensional coordinate space is called the mega distributed lag model, which is alternately called "the infinity coordinates space (specific approach)" (See Figure 1). The application of the infinity coordinates space (specific approach) can help to visualize a large number of

distributed lag functions simultaneously in a single image.

Using a large number of distributed lag functions in different spaces in the infinity coordinates space (specific approach) comes with some prerequisite conditions: first of all, we refer to a single point plotted on an infinite dimensional coordinates space (specific approach) as a "T-dot". Second, two T-dots cannot occupy the same space at the same time. Third, each T-dot refers into its general-space, sub-space and micro-space, respectively. Fourth, different T-dots can represent different intervals or units of time. These times are followed by partial times and constant times, with the type of time being related by the plot of the T-dot to its micro-space. For example, the moment in which the T-dot is initially plotted can be considered partial time, but after plotting the second T-dot, the first T-dot then reverts to constant time. The fifth and final condition is that each micro-space keeps its specific distributed lag function (see Figure 2) according to the general-space and sub-space (see Expression 2.5).

Each distributed lag function in the mega distributed lag model is indexed by time, starting from general-space "0", sub-space "0" and micro-space "0". From there it increases by one unit until we arrive to the general-space "∞", sub-space "∞" and micro-space "∞" (see Figure 2). All lag distributed functions in different micro-spaces under different general-space and sub-space levels apply the *Omnia Mobilis*, "Everything is moving", assumption (Ruiz Estrada, 2011a; 2011b). This assumption allows a large number of variables to be simultaneously observed in our multi-dimensional analysis, as opposed to the *ceteris paribus* assumption, which keeps variables constant. Recall that each general-space is formed by an infinite number of sub-spaces, and each sub-space is formed by an infinite number of micro-spaces, and finally each micro-space has an infinite number of T-dots (see Figures 1 and 2). Usually, the T-dots originate from the JI-variables. If we join each individual T-dot by a straight line into each one's respective micro-space, this creates a scatter graph that shows the trend of any distributed lag function. Additionally, recall that the mega distributed lag model consists of infinitely many micro-spaces, each of which shows different distrib-
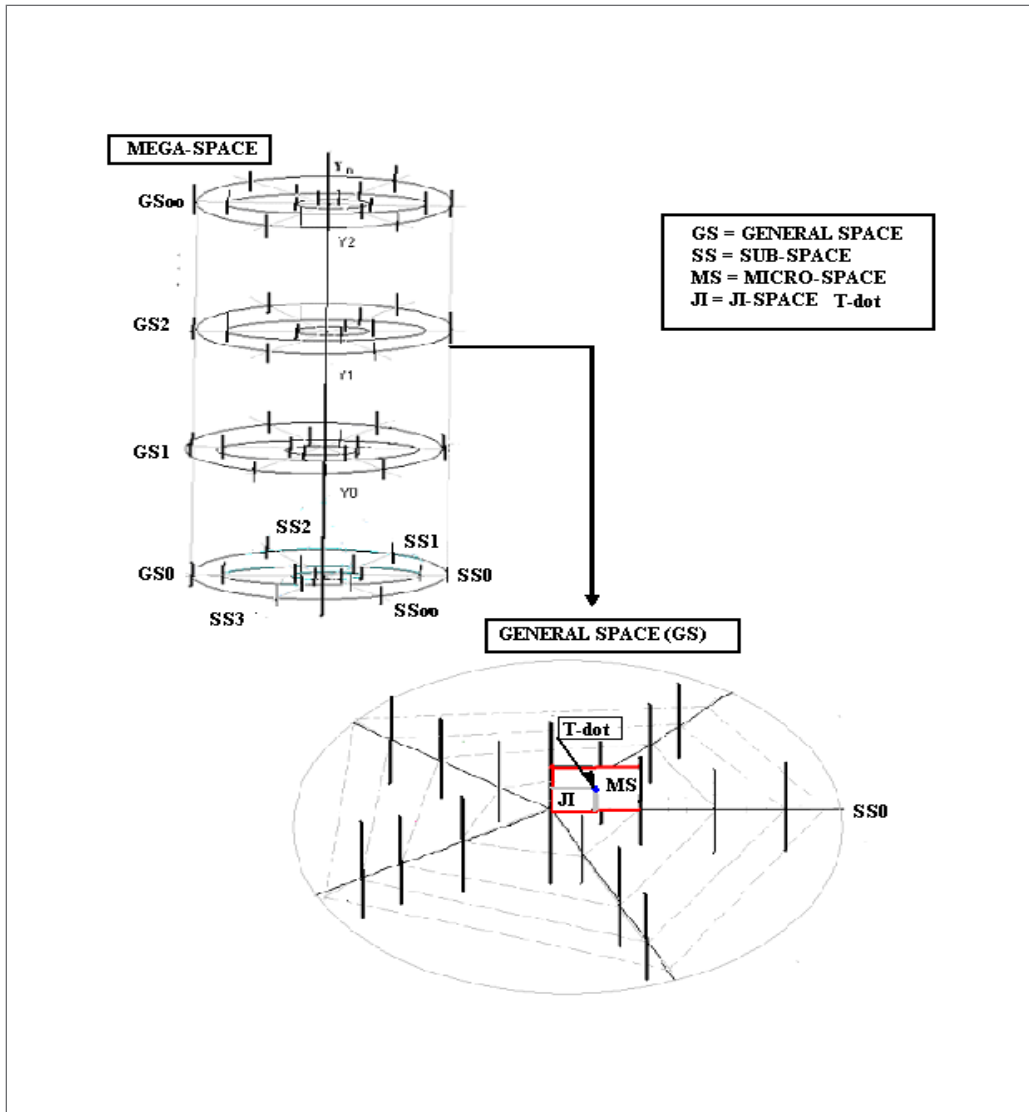
Figure 1. The Infinity Coordinate Space (Specific Approach)

uted lag functions to simultaneously observe multiple trends in full motion (see Figure 2).

The final output of the mega distributed lag model is based on the result of a single determinant (∆) from a matrix with m rows and n columns (see Expression 2.6). This matrix accounts for the

final results of all distributed lag functions from different micro-spaces under different general levels and sub-levels of analysis. The final output (determinant) is labelled "the JIM-coefficient". This JIM-coefficient is a measure encompassing all of the information from the analysis, and so can be
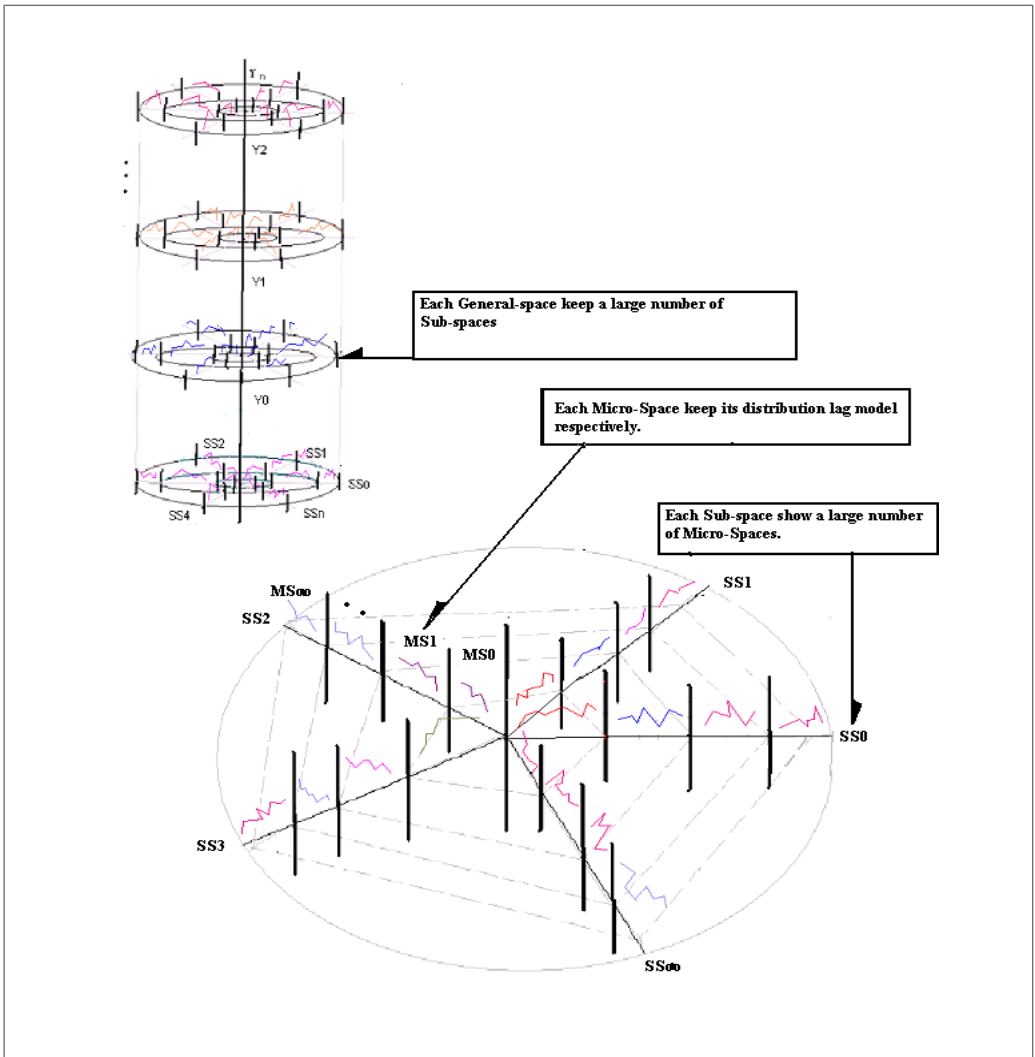
Figure 2. The Mega Distributed Lag Model

considered the final result from the mega distribution lag model.

General-Space "n"

$$Y^n_{tp(GV:SV:MV:JIV)} = \alpha_{(GV:SV:MV:JIV)} + \beta^{L0}_{(GV:SV:MV:JIV)} X^{tp/0}_{(GV:SV:MV:JIV)} +$$
$$+ \beta^{L1}_{(GV:SV:MV:JIV)} X^{tp/1}_{(GV:SV:MV:JIV)-I} + \ldots +$$
$$+ \beta^{L\infty}_{(GV:SV:MV:JIV)} X^{tp/\infty}_{(GV:SV:MV:JIV)-n} + u^{tk}_{(GV:SV:MV:JIV)} \qquad (2.1)$$

Where
$$JIV = \{0, 1, 2 \ldots \infty\}$$
$$n = \{0, 1, 2 \ldots \infty\}$$

Therefore,

$$E / U^{tk} / = K_o \qquad (2.2)$$
$$Var(U^{tk}) = \sigma^i_{(GV:SV:MV:JIV)} \qquad (2.3)$$
$$Cov(U^{tk}, U^{tk}_s) = \sigma^i_{(GV:SV:MV:JIV)} \qquad (2.4)$$

General model

General-Space 0

Sub-Space 0:

$SV_0 = Y^0_{tp(0:0:0)} = \alpha_{(0:0:0)} + \beta^{L0}_{(0:0:0:0)} X^{tp/0}_{(0:0:0:0)} + \beta^{L1}_{(0:0:0:1)} X^{tp/1}_{(0:0:0:1)-1} + \ldots + \beta^{L\infty}_{(0:0:0:\infty)} X^{tp/\infty}_{(0:0:0:0:\infty)-n} + u^{tk}_{(0:0:0)} \circledR \ldots \circledR$

$Y^0_{tp(0:0:1)} = \alpha_{(0:0:1)} + \beta^{L0}_{(0:0:1:0)} X^{tp/0}_{(0:0:1:0)} + \beta^{L1}_{(0:0:1:1)} X^{tp/1}_{(0:0:1:1)-1} + \ldots + \beta^{L\infty}_{(0:0:1:\infty)} X_{tp/\infty(0:0:1:\infty)-n} + u^{tk}_{(0:0:1)} \circledR \ldots \circledR$

$Y^\infty_{tp(0:0:\infty)} = \alpha_{(0:0:\infty)} + \beta^{L0}_{(0:0:\infty:0)} X^{tp/0}_{(0:0:\infty:0)} + \beta^{L1}_{(0:0:\infty:1)} X^{tp/1}_{(0:0:\infty:1)-1} + \ldots + \beta^{L\infty}_{(0:0:\infty:\infty)} X^{tp/\infty}_{(0:0:\infty:\infty)-n} + u^{tk}_{(0:0:\infty)} \ldots$

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Sub-Space ∞:

$SV_\infty = Y^*_{tp(0:\infty:0)} = \alpha_{(0:\infty:0)} + \beta^{L0}_{(0:\infty:0:0)} X^{tp/0}_{(0:\infty:0:0)} + \beta^{L1}_{(0:\infty:0:1)} X^{tp/1}_{(0:\infty:0:1)-1} + \ldots + \beta^{L\infty}_{(0:\infty:0:\infty)} X^{tp/\infty}_{(0:\infty:0:\infty)-n} + u^{tk}_{(0:\infty:0)} \circledR \ldots \circledR$

$Y^0_{tp(0:\infty:1)} = \alpha_{(0:\infty:1)} + \beta^{L0}_{(0:\infty:1:0)} X^{tp/0}_{(0:\infty:1:0)} + \beta^{L1}_{(0:\infty:1:1)} X^{tp/1}_{(0:\infty:1:1)-1} + \ldots + \beta^{L\infty}_{(0:\infty:1:\infty)} X^{tp/\infty}_{(0:\infty:1:\infty)-n} + u^{tk}_{(0:\infty:1)} \circledR \ldots \circledR$

$Y^\infty_{tp(0:\infty:\infty)} = \alpha_{(0:\infty:\infty)} + \beta^{L0}_{(0:\infty:\infty:0)} X^{tp/0}_{(0:\infty:\infty:0)} + \beta^{L1}_{(0:\infty:\infty:1)} X^{tp/1}_{(0:\infty:\infty:1)-1} + \ldots + \beta^{L\infty}_{(0:\infty:\infty:\infty)} X^{tp/\infty}_{(0:\infty:\infty:\infty)-n} + u^{tk}_{(0:\infty:\infty)}$

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General-Space ∞

Sub-Space 0:

$SV_0 = Y^*_{tp(\infty:0:0)} = \alpha_{(\infty:0:0)} + \beta^{L0}_{(\infty:0:0:0)} X^{tp/0}_{(\infty:0:0:0)} + \beta^{L1}_{(\infty:0:0:1)} X^{tp/1}_{(\infty:0:0:1)-1} + \ldots + \beta^{L\infty}_{(\infty:0:0:\infty)} X^{tp/\infty}_{(\infty:0:0:0:\infty)-n} + u^{tk}_{(\infty:0:0)} \circledR \ldots \circledR$

$Y^0_{tp(\infty:1:1)} = \alpha_{(\infty:1:1)} + \beta^{L0}_{(\infty:1:1:0)} X^{tp/0}_{(\infty:1:1:0)} + \beta^{L1}_{(\infty:1:1:1)} X^{tp/1}_{(\infty:1:1:1)-1} + \ldots + \beta^{L\infty}_{(\infty:1:1:\infty)} X^{tp/\infty}_{(\infty:1:1:\infty)-n} + u^{tk}_{(\infty:1:1)} \circledR \ldots \circledR$

$Y^\infty_{tp(\infty:\infty:\infty)} = \alpha_{(\infty:\infty:\infty)} + \beta^{L0}_{(\infty:\infty:\infty:0)} X^{tp/0}_{(\infty:\infty:\infty:0)} + \beta^{L1}_{(\infty:\infty:\infty:1)} X^{tp/1}_{(\infty:\infty:\infty:1)-1} + \ldots + \beta^{L\infty}_{(\infty:\infty:\infty:\infty)} X^{tp/\infty}_{(\infty:\infty:\infty:\infty)-n} + u^{tk}_{(\infty:\infty:\infty)} \ldots$

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Sub-Space ∞:

$SV_\infty = Y^*_{tp(\infty:\infty:0)} = \alpha_{(\infty:\infty:0)} + \beta^{L0}_{(\infty:\infty:0:0)} X^{tp/0}_{(\infty:\infty:0:0)} + \beta^{L1}_{(\infty:\infty:0:1)} X^{tp/1}_{(\infty:\infty:0:1)-1} + \ldots + \beta^{L\infty}_{(\infty:\infty:0:\infty)} X^{tp/\infty}_{(\infty:\infty:0:\infty)-n} + u^{tk}_{(\infty:\infty:0)} \circledR \ldots \circledR$

$Y^0_{tp(\infty:\infty:1)} = \alpha_{(\infty:\infty:1)} + \beta^{L0}_{(\infty:\infty:1:0)} X^{tp/0}_{(\infty:\infty:1:0)} + \beta^{L1}_{(\infty:\infty:1:1)} X^{tp/1}_{(\infty:\infty:1:1)-1} + \ldots + \beta^{L\infty}_{(\infty:\infty:1:\infty)} X^{tp/\infty}_{(\infty:\infty:1:\infty)-n} + u^{tk}_{(\infty:\infty:1)} \circledR \ldots \circledR$

$Y^\infty_{tp(\infty:\infty:\infty)} = \alpha_{(\infty:\infty:\infty)} + \beta^{L0}_{(\infty:\infty:\infty:0)} X^{tp/0}_{(\infty:\infty:\infty:0)} + \beta^{L1}_{(\infty:\infty:\infty:1)} X^{tp/1}_{(\infty:\infty:\infty:1)-1} + \ldots + \beta^{L\infty}_{(\infty:\infty:\infty:\infty)} X^{tp/\infty}_{(\infty:\infty:\infty:\infty)-n} + u^{tk}_{(\infty:\infty:\infty)}$

(2.5.)

$$\Delta = \begin{pmatrix} GV_0 = SV_0 \circledR SV_1 \circledR \ldots \circledR SV_{\infty\ldots} \\ . \qquad\qquad\qquad . \\ . \qquad\qquad\qquad . \\ . \qquad\qquad\qquad . \\ GV\infty = SV_0 \circledR SV_1 \circledR \ldots \circledR SV_{\infty\ldots} \end{pmatrix}$$ (2.6.)

Variables:

GV = General variable, SV = Sub-variable, MS = Micro-variable, JIV= JI-variable, tp = Partial Time, tk = Constant Time, ® = Window refraction

3. Concluding Remarks

In conclusion, previous attempts at graphing multi-dimensional behavior in the distributed lag model have stopped at two dimensions. Because of this, we advocate the use of an alternative general distributed lag model, which we call "the mega distributed lag model". This paper has developed the framework for extending those representations of distributed lag models to an arbitrarily large number of dimensions. The advantage of doing so is to aid in the identification of abnormally strong relationships between the lagged variables and the dependent variable, which two-dimensional figures overwhelmingly fail to capture. Multi-Dimensional Coordinate Space framework treats each dimension uniformly at the cost of coarse representation (Ruiz Estrada, 2011b), enabling us to delve deeper into understanding the behavior of an infinite distributed lag model. Hence, the 2-dimensional Cartesian coordinate plane and the 3-dimensional coordinate space are limited in their ability to visualize a large number of variables simultaneously in the same graphical space and time framework.

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Appendix A

Consider the autoregressive distributed lag model of order p and q, ADL (p, q), defined for a scalar variable y_t as

$$y_t = \alpha + \sum_{i=1}^{p} \gamma_i\, y_{t-i} + \sum_{i=0}^{q} \beta_i x_{t-i} + \eta_t \qquad (A1)$$

where η_t is a scalar independent white noise and x_t is a K-dimensional column stationary variable vector process. The sequence $\{\eta_t\}$ is a white-noise process if for each period t,

$$E(\eta_t) = E(\eta_{t-1}) = \ldots = 0$$

$$E(\eta_t{}^2) = E(\eta_{t-1}{}^2) = \ldots = \sigma^2$$

$$E(\eta_t \eta_{t-s}) = E(\eta_{t-j}\eta_{t-j-s}) = 0 \text{ for all } \eta$$

As long as the error term η_t is a white noise process, the ADL model can be estimated consistently by ordinary least squares. Using the lag operator L applied to each component of a vector, $L^k x_t = x_{t-k}$ (lag of a constant: $L\alpha = \alpha$), it is convenient to define the lag polynomial $\gamma(L)$ and the vector polynomial $\beta(L)$

$$a(L) = \alpha$$

$$\gamma(L) = (1 + \gamma_1 L + \gamma_2 L^2 + \ldots + \gamma L^p)$$

$$\beta(L) = (\beta_0 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q)$$

Rearranging the lag polynomial $\gamma(L)y_t$, equation (A1) takes the straightforward form

$$\gamma(L)y_t = a + \beta(L)x_t + \eta_t \qquad (A2)$$

Equation (A2) implies that the current value of y depends on all previous values of a, x, and η. The dynamic effects correspond to the lag weights of the infinite moving average representation

$$\frac{\partial y_{t+i}}{\partial x_t} = \frac{\partial y_t}{\partial x_{t-i}} = \beta_i \qquad (A3)$$

Note that the first equation in (A3) requires that the time-series relationship between y and x be stationary, so we can think of β_i either as the effect of current x_t on future y_{t+i} or as the effect of past x_{t-i}

on current y_t. In order to obtain stability, it is maintained that

$$\gamma(\varphi) = 0 \implies |\varphi| > 1 \,\forall \varphi \in N \qquad (A4)$$

Under this condition there exists an absolutely summable infinite expansion of the inverted polynomial $\gamma^{-1}(L)$:

$$\gamma^{-1}(L) = \frac{1}{\gamma(L)} = \sum_{j=0}^{\infty} \gamma_j^* L^j, \qquad \sum_{j=0}^{\infty} |\gamma_j^*| < \infty \qquad (A5)$$

Invertibility of γ(L) hence yields the following representation:

$$y_t = \frac{\beta(L)}{\gamma(L)} x_t + \psi_t, \qquad \gamma(L)\psi_t = \eta_t \qquad (A6)$$

where η_t has a stable autoregressive structure of order p. Expanding $\gamma^{-1}(L)$ provides an infinite distributed lag representation,

$$y_t = \left(\sum_{j=0}^{\infty} \gamma_j^* L^j \right) \left(\sum_{j=0}^{q} \beta_j^* L^j \right)' x_t + \psi_t = \sum_{j=0}^{\infty} \omega_j' x_{t-j} + \psi_t \qquad (A7)$$

where ω_j are the vectors of dynamic multipliers derived by the method of indetermined coefficients. The vector of long-run multipliers of the ADL (p, n) model may therefore be easily computed from:

$$\xi = \frac{\beta(1)}{\gamma(1)} = \sum_{j=0}^{\infty} \omega_j \qquad (A8)$$

By re-arranging the x's one obtains with $\Delta = 1 - L$:

$$y_t = \sum_{i=1}^{p} \gamma_i\, y_{t-i} + \gamma(1)\,\xi' x_t - \sum_{i=0}^{q-1} \left(\sum_{j=i+1}^{q} \beta_j \right)' \Delta x_{t-i} + \psi_t \qquad (A9)$$

where y_t is related to its own past, to contemporaneous x_t and differences Δx_{t-i}. This is called the error correction model (ECM).